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AN INNOVATIVE SOLUTION FOR THE PROBLEM OF BLOOD FLOW THROUGH STENOSED ARTERY USING GENERALIZED BINGHAM PLASTIC FLUID MODEL

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ABSTRACT

This mathematical model has been presented to study the effect of a stenosis shape on arterial blood flow characteristics with the representation of blood by Bingham plastic fluid model. The governing equations of proposed model are solved and closed form expressions for the blood flow characteristics, namely dimensionless resistance to flow, flow rate and wall shear stress are derived. It has been found that the wall shear stress and resistance to flow increase with increasing tube radius for constant value of the stenosis height, while decreases as stenosis shape parameter increases.

KEYWORDS: Blood Flow, Bingham Plastic Fluid, Resistance to Flow, Wall Shear Stress, Stenosis Shape Parameter

INTRODUCTION

Atherosclerosis (ath-er-o-skler-O-sis) is a disease in which plaque (plak) builds up inside arteries. Arteries are blood vessels that carry oxygen-rich blood to heart and other parts of the body. Plaque is made up of fat, cholesterol, calcium, and other substances found in the blood. Over time, plaque hardens and narrows arteries. This limits the flow of oxygen-rich blood to organs and other parts of body. Atherosclerosis can lead to serious problems, including heart attack, stroke, or even death. The study of blood flow through mammalian circulatory system has been the subject of scientific research for about a couple of centuries. Like most of the problems of blood flow, it is complex one due to the complicated structure of blood, the circulatory system and their constituent materials. The experimental studies and the theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes and practical applications. It has been reported that the fluid dynamical properties of blood flow through non-uniform cross section of the arteries play a major role in the fundamental understanding and treatment of many cardiovascular diseases. Several researchers have studied the blood flow characteristics due to the presence of a stenosis in the tapered arteries. Blood behaves like a Newtonian fluid when it flows through larger arteries at high shear rates, whereas it behaves like a non-Newtonian fluid when it flows through narrow arteries at low shear rates. In the region of narrowing arterial constriction, the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis. The possibility that the haemodynamic factors play an important role in the genesis and proliferation of stenosis has attracted the interest of researchers to study blood flow through local constrictions Young (1968); Young and Tsai (1973); Deshpande et al. (1976), Caro et al. (1978); Ahmed and Giddens (1983); Ku (1997) and others during the past few decades. An account of the most of the theoretical and experimental studies, reported so far, may be had from Young (1979), Srivastava (1996, 2002), Sarkar and Jayaraman (1998), Mishra and Verma (2007), Mekheimer and Kot (2008), Srivastava and Rastogi (2009, 2010), etc. The analysis of blood flow through a symmetrically stenosed artery has been studied by Singh et al. (2009). Sanyal and Maji (1999) investigated the unsteady blood flow through an indented tube in presence of stenosis. Chakravarty and Datta (1990) performed rheological study on the effect of mild stenoses on the flow behavior of blood in a stenosed arterial segment. The various geometries of stenosis have been suggested by the researchers.

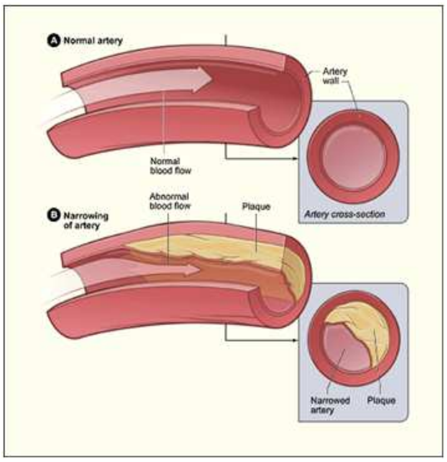

Figure 1: (A) Shows a Normal Artery with Normal Blood Flow
Figure (B) Shows an Artery with Plaque Buildup

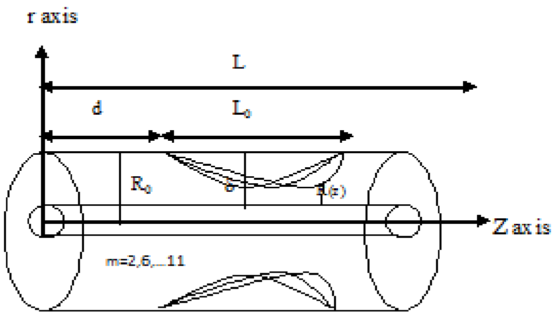

Figure 2: Stenotic Artery

The cosine-shaped geometry was considered and analyzed with different parameters by many researchers like Young (1968), Kapur (1985), Chakravarty (1987). The power-law and Casson fluid models with cosine-shaped geometry were discussed by Shukla et al. (1980). A composite shaped geometry of arterial stenosis was suggested and investigated by Mekheimer (2008). The bell-shaped geometry with different fluids was discussed by Misra and Shit (2006). In all of the above studies the shape of stenosis was considered to be symmetrical about the axis as well as radius of the flow cylinder.

The radially nonsymmetric stenosis has been analyzed by Sanyal and Maji (1999), Srivastava and Saxena (1999), Srivastava (1996). The effects of shape of stenosis on the resistance to blood flow through an artery has been investigated by Haldar (1985). Due to the presence of a new parameter the formulation of our model is mathematically more general and includes the model of Haldar (1985) as a special case.

In the present mathematical model, we have studied a problem in which blood flow has been considered axially non-symmetric but radially symmetric with mild stenosed artery by introducing blood as Bingham plastic fluid model. The effects of stenosis shape parameter on resistance to flow, apparent viscosity, stenosis size, yield stress, and stenosis length, have investigated. The schematic diagram of the flow is given by Figure.1 and Figure.2.

Formulation of the Mathematical Model

We have considered an artery having mild stenosis. The flow of blood is assumed to be steady, laminar and fully-developed. Blood is taken as a Bingham plastic fluid. It is assumed that stenosis is symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. The mathematical expression for geometry can be written as,

$$\left.\begin{aligned}\frac{R(z)}{R_0} &= 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], & d \le z \le d + L_0 \\ &= 1, & \text{otherwise,}\end{aligned}\right\} \tag{1}$$

$$A = \frac{\delta}{R_0 L_0^m}\frac{m^{m/(m-1)}}{(m-1)},$$

where

R_0 : Radius of normal tube

R(z) : Radius of stenotic region

L : The length of the artery

L_0 : The length of the stenosis

d : Distance between equispaced points

δ : Maximum height of stenosis ($\delta << R_0$)

m : Parameter determining the shape of stenosis ($m \ge 2$)

Conservation Equation and Boundary Conditions

The equation of motion for laminar and incompressible, steady, fully-developed, one-dimensional flow of blood whose viscosity varies along radial direction in an artery reduces to:

$$\left.\begin{aligned}0 &= -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau)}{\partial z}, \\ 0 &= -\frac{\partial P}{\partial r},\end{aligned}\right\} \tag{2}$$

where (z, r) are co-ordinates with z measured along the axis and r measured normal to the axis of the artery.

The boundary conditions are introduced to solve the above equations,

$$\left.\begin{aligned}&\partial u/\partial r = 0 \quad \text{at } r = 0, \quad u = 0 \quad \text{at } r = R(z) \\ &\tau \text{ is finite} \quad \text{at } r = 0 \\ &P = P_0 \quad \text{at } z = 0, \quad P = P_L \quad \text{at } z = L\end{aligned}\right\} \tag{3}$$

Bingham Plastic Fluid Model

For Bingham plastic fluid, the stress-strain relation is given by

$$\tau = \tau_0 + \mu\left(-\frac{du}{dr}\right)$$

$$\text{where} \quad \tau = \left(-\frac{dp}{dz}\frac{r}{2}\right), \; \tau_0 = \left(-\frac{dp}{dz}\frac{R_P}{2}\right), \tag{4}$$

u : axial velocity

μ : viscosity of fluid

(-dp/dz) : pressure gradient

Solution of the Problem

The expression for the velocity, u obtained as the solution of equation (2) subject to the boundary conditions (3) and equation (4), is obtained as (for $R_P \leq r \leq R(z)$)

$$u=-\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2-\left(\frac{r}{R_0}\right)^2\right]+\frac{\tau_0 R_0}{\mu}\left[\left(\frac{R}{R_0}\right)-\left(\frac{r}{R_0}\right)\right]-\frac{4R_0^{3/2}\tau_0}{3\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)^{1/2}\left[\left(\frac{R}{R_0}\right)^{3/2}-\left(\frac{r}{R_0}\right)^{3/2}\right] \quad (5)$$

The constant plug flow velocity, u_P may be obtained from equation (5) evaluated at $r = R_P$.

The volumetric flow rate Q can be defined as,

$$Q=\int_0^R 2\pi u r\,dr=\pi\int_0^R r\left(-\frac{du}{dr}\right)dr, \quad (6)$$

The flow flux, Q when Rp<< R (i.e., the radius of the plug flow region is very small as compared to the non-plug flow region), is calculated as

$$Q=-\frac{R_0^4\pi}{8\mu}\frac{dp}{dz}\left(\frac{R}{R_0}\right)^4+-\frac{\tau_0\pi}{3\mu}\left(\frac{R}{R_0}\right)^3+-\frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\frac{R}{R_0}\right)^7\right\}^{1/2} \quad (7)$$

$$Q=\frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right)f(\bar{y}), \quad (8)$$

From above equation pressure gradient is written as follows,

$$\left(-\frac{dp}{dz}\right)=\frac{8\mu Q}{\pi R_0^4}f(\bar{y}) \quad (9)$$

$$f(\bar{y})=\left(\bar{y}\right)^4+-\frac{\tau_0\pi}{3\mu}\left(\bar{y}\right)^3+-\frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\bar{y}\right)^7\right\}$$

Integrating equation (9) using the condition (3) $P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$. We have

$$\Delta P = P_L - P_0 = \frac{8\mu Q}{\pi R_0^4}\int_0^L \frac{dz}{\left(R(z)/R_0\right)^4 f(\bar{y}(z))} \quad (10)$$

The resistance to flow is denoted by λ and defined as follows,

$$\lambda = \frac{P_L - P_0}{Q} \quad (11)$$

The resistance to flow from equation (11) using equations (10) is written as,

$$\lambda = 1 - \left(L_0 / L\right) + \left(f_0 / L\right) \int_{d}^{d+L_0} \frac{dz}{(R(z)/R_0)^4 f(\overline{y}(z))} \tag{12}$$

where f_0 is given by $f_0 = \left(R/R_0\right)^4 + - \frac{\tau_0 \pi}{3\mu}\left(R/R_0\right)^3 + - \frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)\left(R/R_0\right)^7\right\}$

Following the apparent viscosity (μ_{app}) is defined as follows;

$$\mu_{app} = \frac{1}{\left(R(z)/R_0\right)^4 f(\overline{y})} \tag{13}$$

The shearing stress at the wall can be defined as;

$$\tau_R = \tau_0 + \mu\left(-\frac{du}{dr}\right)_{r=R(z)} \tag{14}$$

RESULTS AND DISCUSSIONS

In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, apparent viscosity and wall shear stress for normal and diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Figure 3-7 by using the values of parameter based on experimental data in artery.

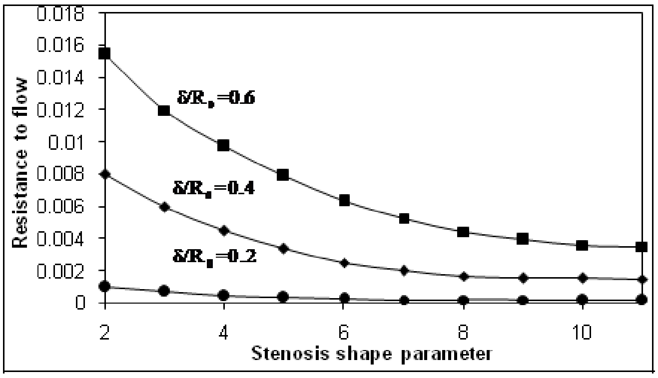

Figure 3: Variatuin of Resistance to Flow with m

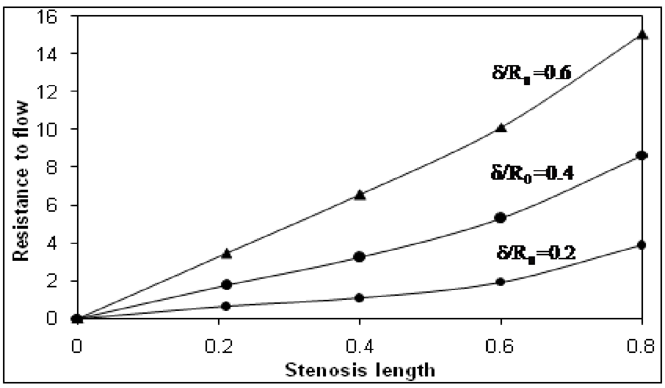

Figure 4: Variation of Resistance to Flow with Stenosis Length

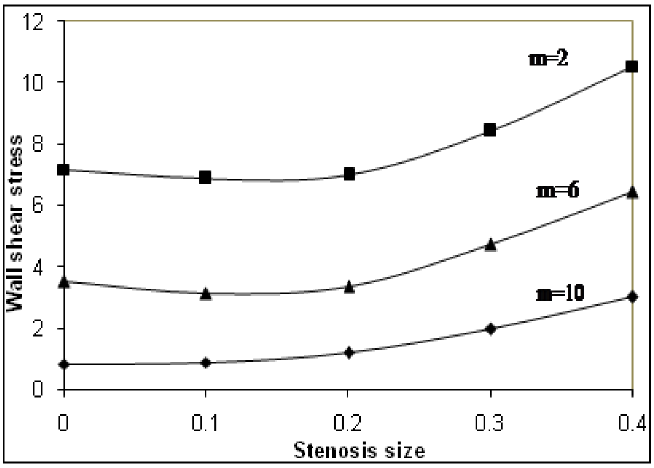

Figure 5: Variation of Wall Shear Stress with Stenosis Size for different Values of m

Figure 3 reveals the variation of resistance to flow (λ) with stenosis shape parameter (m) for different values of stenosis size (δ/R_0). It is observed that the resistance to flow (λ) decreases as stenosis shape parameter (m) increases and maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. It has also been seen from this graph that resistance to flow (λ) increases as stenosis size (δ/R_0) increases.

These results are therefore consisting to the result of Mishra and Verma (2007). In Figure 4 the variation of resistance to flow (λ) with stenosis length (L_0/L) for different values of stenosis size (δ/R_0) has been shown. This figure depicts that the resistance to flow (λ) increases as stenosis size (δ/R_0) and stenosis length (L_0/L) increases.

These results are similar to the results of Srivastava (1999). Figure 5 shows the variation of wall shear stress (τ) with stenosis size for different values of stenosis shape parameter (m). It may be observed from the figure that the wall shear stress (τ) increases as stenosis size increases while decreases as stenosis shape parameter (m) increases.

These results are consistent to the observation of Chakravarty (1987). Figure 6 shows the variation of wall shear stress (τ) with stenosis size for different values of stenosis length (L_0/L). It is clear from the figure that the wall shear stress (τ) increases as stenosis size and stenosis length increases. These results are consistent to the observation of Haldar (1985). The variation of apparent viscosity with stenosis length (L_0/L) for different values of stenosis size (δ/R_0) has been depicted in Figure 7. This figure shows that the of apparent viscosity increases as stenosis size (δ/R_0) increases. This result is similar to the results of Sanyal and Maji (1999).

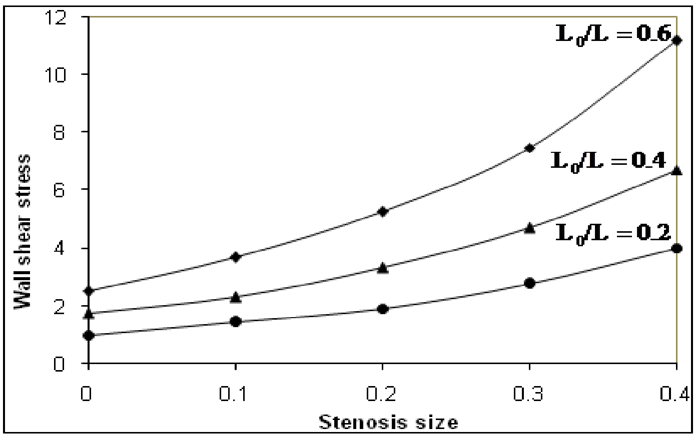

Figure 6: Variation of Wall Shear Stress with Stenosis Size for Different Values of Stenosis Length

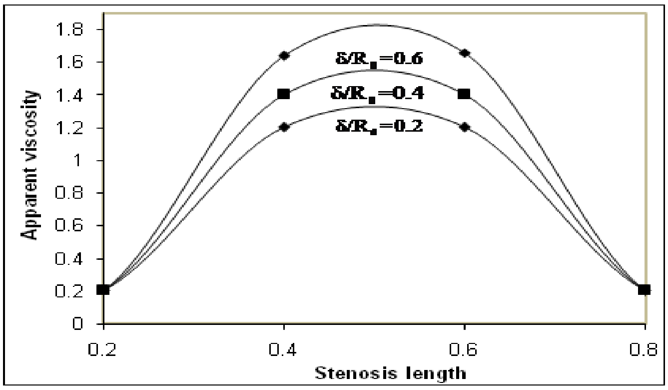

Figure 7: Variation of Apparent Viscosity with Stenosis Length

Concluding Remarks

In his paper, we have studied the effect of stenosis shape parameter on resistance to blood flow, wall shear stress and apparent viscosity in an artery by introducing blood as Bingham plastic fluid model. It has been concluded that the resistance to blood flow, wall shear stress and apparent viscosity increases as stenosis size and stenosis length increases while decreases as stenosis shape parameter increases. So it has shown that the results were greatly influenced by the change of stenosis shape parameter. In an artery flow, the viscosity of blood found to vary with the arterial radius decreasing with it. This model helps for the people working in the field of physiological fluid dynamics as well as to the medical practitioners.

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